DESCRIPTION OF MASS-EXCHANGE PROCESSES IN POROUS MEDIA AT LOW VALUES OF THE PECLET NUMBER

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The small perturbation method is used to construct equations for two approximations with respect to the Peclet number for the problem of mass exchange in a porous medium with mass sources.

To describe certain heat–mass exchange processes in porous media such as adsorption, washing of soils and deposits, and heat transfer in heterogeneous media a mathematical model is used. Such a model assumes the existence of two phases, one of which determines the matter concentration in the flow portion of the porous body, while the second defines the concentration in stagnant regions. Equations for such processes have been derived several times [1-4] by various methods. We will use the equations in the form

$$\mathrm{Pe}\,[\partial c/\partial\tau + \varphi\partial a/\partial\tau + \partial c/\partial z - w\,(a,\ c)] = \partial^2 c/\partial z^2, \tag{1}$$

$$\mathrm{Pe}\,\varphi\,\partial a/\partial\tau = n\,[c - f\,(a)], \tag{2}$$

usually employed in adsorption theory [1], and consider a possible chemical reaction, in the general case dependent on the concentrations a and c. Here $\tau=tu/l\varepsilon$, $n=kl^2/D$, $\varphi=(1-\varepsilon)/\varepsilon$, , and the factors before the functions f and w which appear upon dedimensionalization will be included within those functions themselves. We will call the function $a = f^{-1}(c)$ the sorption isotherm. The right side of Eq. (2) represents a deviation from equilibrium characterizing the isotherm, and is the moving force which produces a change in the concentration a toward equilibrium, which is described by Eq. (2). The right side of Eq. (1) defines the change in concentration in the flow region due to longitudinal diffusion, which is balanced on the left side by a corresponding transient term, the influx (efflux) of matter from stagnant zones ($\varphi\partial a/\partial\tau$), convective transport ($\partial c/\partial z$) and change in concentration due to chemical conversion w.

It is known [5] that to describe heat–mass exchange in porous bodies the coefficients of the model can be considered constants only at some time after the commencement of the process, although the problem with constant D and k for the entire time interval is widely used in practical applications and in many cases conveys the pattern of heat–mass exchange properly. We will therefore assume D = const and k = const. In some cases, for example, in treating washing of precipitates [3] the presence of material in a third phase is considered — on precipitate particles or the boundaries between stagnant zones and the flow region. In this case a third equation of the form of Eq. (2) and corresponding terms in Eqs. (1) and (2) are usually added. The presence of a third phase will not be considered here, since the overall pattern of constructing a solution remains unchanged; however, the expressions become more cumbersome.

The boundary conditions for Eq. (1) will be taken in the form

$$\left.\frac{\partial c}{\partial z}\right|_{z=0} = \mathrm{Pe}\,[c - R\,(\tau)],\quad \left.\frac{\partial c}{\partial z}\right|_{z=1} = 0, \tag{3}$$

where the function $R(\tau)$ characterizes the flow of material entering the porous region of the porous body through the initial section. The second condition, Eq. (3) (at the output), is the conventional Dankwerst condition. We also specify initial conditions in the quite general form

$$c|_{\tau=0} = K\,(z),\ a|_{\tau=0} = L\,(z). \tag{4}$$

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To construct two approximations with respect to the small parameter Pe it is necessary that the function R be differentiable. We will also assume that the function f is differentiable and monotonically increasing, i.e., $f'(a) > 0$, and thus has a reciprocal function f^{-1}. We also assume $f(0) = 0$.

It should be noted that the smallness of the parameter Pe makes numerical solution of the problem of Eqs. (1)-(4) difficult, since a very small step in τ, is required, and to reach times of the order $t \sim l/u$, at which the changes in concentrations a and c most interesting in practice occur, requires a hugh number of steps. Computation errors then become unavoidable, so that the perturbation method becomes very useful in such problems.

We will seek a solution of Eqs. (1)-(4) in the form of the following expansions:

$$c = f_0(z, \tau) + \mathrm{Pe} f_1(z, \tau) + \ldots, \quad a = F_0(z, \tau) + \mathrm{Pe} F_1(z, \tau) + \ldots, \tag{5}$$

which when substituted in the equations and boundary conditions, after terms of like order in Pe are grouped, produce the equations

$$\partial^2 f_0 / \partial z^2 = 0, \quad f_0 = f(F_0), \tag{6}$$

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{\partial f_0}{\partial \tau} + \varphi \frac{\partial F_0}{\partial \tau} + \frac{\partial f_0}{\partial z} - w(F_0, f_0), \; n\,[f_1 - F_1 f'(F_0)] = \varphi \frac{\partial F_0}{\partial \tau}, \tag{7}$$

$$\frac{\partial^2 f_2}{\partial z^2} = \frac{\partial f_1}{\partial \tau} + \varphi \frac{\partial F_1}{\partial \tau} + \frac{\partial f_1}{\partial z} - \left.\frac{\partial w}{\partial a}\right|_{a=F_0;c=f_0} F_1 - \left.\frac{\partial w}{\partial c}\right|_{a=F_0;c=f_0} f_1 \tag{8}$$

and boundary equations

$$\frac{\partial f_0}{\partial z} = 0, \quad \frac{\partial f_1}{\partial z} = f_0 - R, \quad \frac{\partial f_2}{\partial z} = f_1 \text{ at } z = 0, \tag{9}$$

$$\partial f_i / \partial z = 0, \; i = 0, 1, 2, \text{ at } z = 1, \tag{10}$$

which generalize the previously obtained expressions of [6] to the nonlinear case. The problem of Eqs. (6)-(10) is singular-perturbed [7, 8]. In order to obtain the initial conditions for the problem, their solution must be merged with the solution of the internal problem (17)-(22), which describes the solution of the general problem at times of the order of τ/Pe [6].

Equation (6) and boundary conditions (9), (10) indicate that the function f_0 is dependent solely on $\tau: f_0 = A(\tau)$, while F_0 is related to f_0 by the equilibrium sorption equation $F_0(\tau) = f^{-1}[A(\tau)]$. With consideration of this fact, from Eq. (7) we find

$$f_1 = (A' - w + \varphi F_0')(z - 1)^2/2 + B(\tau), \; n\,[f_1 - f'(F_0) F_1] = \varphi F_0', \tag{11}$$

where $A(\tau)$ and $B(\tau)$ are functions of time yet to be determined. From the first expression of Eq. (11) and the boundary condition for the function f_1 at $z = 0$, we find the equation for definition of the function $A(\tau)$:

$$A'(\tau) + A(\tau) + \varphi\, \{f^{-1}[A(\tau)]\}'_\tau = w\{f^{-1}[A(\tau)], A(\tau)\} + R(\tau). \tag{12}$$

We obtain an equation for the function $B(\tau)$ by integration of Eq. (8) between the limits [0, 1] with use of previously obtained relationships (11), (12) and boundary conditions (9), (10):

$$B' + B + \frac{1}{6}(R' - A') + \varphi \frac{d}{d\tau}\left\{[B + \frac{1}{6}(R - A) - \varphi F_0'(\tau)/n] / f_a'\right\} = \tag{13}$$

$$\frac{\partial w}{\partial c}\left[B + \frac{1}{6}(R - A)\right] + \frac{\partial w}{\partial a}\left[B + \frac{1}{6}(R - A) - \varphi F_0'(\tau)/n\right] / f_a'.$$

We note that Eq. (13) is linear in the function $B(\tau)$:

Thus, we have obtained ordinary differential equations from which we can find the functions A and B and thus obtain two approximations in the Peclet number for the problem of Eqs. (1)-(4) for the scales of practical interest, of the order of l/u. It remains only to find the initial conditions for Eqs. (12), (13). To do this, we made use of certain characteristics of the internal solution. We rewrite Eqs. (1), (2), introducing the "internal" time $T = \tau/\mathrm{Pe}$:

$$\partial c/\partial T + \varphi \partial a/\partial T + \mathrm{Pe}\,[\partial c/\partial z - w\,(a,\ c)] = \partial^2 c/\partial z^2, \tag{14}$$

$$\varphi \partial a\,/\,\partial T = n\,[c - f\,(a)]. \tag{15}$$

We seek a solution to the problem of Eqs. (14), (15), (3), (4) in the form of expansions

$$c = g_0\,(z,\ T) + \mathrm{Pe}\,g_1\,(z,\ T) + \ldots,\quad a = G_0\,(z,\ T) + \mathrm{Pe}\,G_1(z,\ T) + \ldots, \tag{16}$$

which when substituted in the equations and boundary conditions produce for the functions g_i, G_i, $i = 0, 1$, the problem

$$\partial g_0\,/\,\partial T + \varphi \partial G_0\,/\,\partial T = \partial^2 g_0\,/\,\partial z^2, \tag{17}$$

$$\varphi \partial G_0\,/\,\partial T = n\,[g_0 - f\,(G_0)], \tag{18}$$

$$\partial g_0\,/\,\partial z|_{z=0;1} = 0, \tag{19}$$

$$\partial g_1\,/\,\partial T + \varphi \partial G_1\,/\,\partial T + \partial g_0\,/\,\partial z = \partial^2 g_1\,/\,\partial z^2 + w\,(g_0,\ G_0), \tag{20}$$

$$\varphi \partial G_1\,/\,\partial T = n\,[g_1 - G_1 f'\,(G_0)], \tag{21}$$

$$\partial g_1\,/\,\partial z|_{z=0} = g_0 - R\,(0),\ \partial g_1\,/\,\partial z\,|_{z=1} = 0. \tag{22}$$

The initial conditions for the problem of Eqs. (17)-(19) will be the original Eq. (4), while for the problem of Eqs. (20)-(22) we have zero initial conditions. We first integrate Eq. (17) over z with limits [0, 1], and then over T with limits [0, T]. Using condition (19) and the initial conditions, we obtain

$$\int\limits_0^1 [g_0 + \varphi G_0]\,dz = \int\limits_0^1 [K\,(z) + \varphi L\,(z)]dz = K^* + \varphi L^* = M = \mathrm{const}. \tag{23}$$

From the principle of limiting merger [8] we have $g_0\,(z,\ \infty) = f_0\,(z,\ 0) = A\,(0) = \mathrm{const}\,(z)$; $G_0\,(z,\ \infty) = F_0\,(0) = \mathrm{const}\,(z)$. Performing the limiting transition $T \to \infty$ in Eq. (23) and using the relationship $f[F_0\,(0)] = A(0)$, we arrive at the equation

$$\varphi f^{-1}\,[A\,(0)] = M - A\,(0) \tag{24}$$

for definition of the initial condition A(0). With the conditions imposed on the function f Eq. (24) has a single root in the interval $A\,(0) \in (0, M)$ which can be found, for example, graphically. A similar approach is used for finding B(0), with Eq. (23) replaced by the expression

$$\begin{gathered}\int\limits_0^1 [g_1 + \varphi G_1 - \frac{z^2}{2}\,(g_0 + \varphi G_0)]\,dz = TR\,(0) - \int\limits_0^1 \frac{z^2}{2}\,[K\,(z) + \varphi L\,(z)]\,dz + \\ + \int\limits_0^T d\xi \int\limits_0^1 \{w\,[G_0\,(z,\ \xi),\ g_0\,(z,\ \xi)] - g_0\,(z,\ \xi)\}\,dz.\end{gathered} \tag{25}$$

Substituting the expressions obtained for f_0 and f_1 in Eq. (5) and rewriting them in internal variables, we find

$$c \sim A\,(0) + \mathrm{Pe}\,\{TA'\,(0) + [R\,(0) - A\,(0)]\,(z - 1)^2/2 + B\,(0)\} + \ldots, \tag{26}$$

which in view of the merger condition gives an asymptotic expansion for the functions g_0, g_1. There is a similar expansion for the function a. Using this together with Eq. (26), we obtain from Eq. (25) in the limit T → ∞ an expression for definition of the initial condition B(0):

$$B(0)=\frac{A(0)-R(0)}{6}+\left\{M+\int_0^\infty dT\int_0^1[w\{G_0(z,T),g_0(z,T)\}-w\{F_0(0),A(0)\}+\right.$$

$$\left.+A(0)-g_0(z,T)]\,dz-\int_0^1\frac{z^2}{2}[K(z)+\varphi L(z)]\,dz+\frac{\varphi^2F_0'(0)}{nf'[F_0(0)]}\right\}\Big/\left\{1+\frac{\varphi}{f'[F_0(0)]}\right\}. \tag{27}$$

Unfortunately, it is only at w = c that B(0) can be determined without solution of the internal problem (17)-(19), (4), which is nonlinear in general form. However, we require only the integral characteristic of the function g_0, which simplifies the problem somewhat, since when approximate methods are used it is often the case that the integral characteristics can be obtained more precisely than local ones. If the function f is linear, f = ma/n, then the solution of the problem of Eqs. (17)-(19), (4) can be represented in the form of series

$$g_0=s_0+\sum_{k=1}^{\infty}s_k(T)\cos(\pi kz),\qquad G_0=S_0+\sum_{k=1}^{\infty}S_k(T)\cos(\pi kz), \tag{28}$$

where

$$S_0=\frac{(mL^*-nk^*)\exp[-T(m/\varphi+n)]+nM}{m+\varphi n},\qquad s_0=M-\varphi S_0, \tag{29}$$

and the functions $S_k(T)$, $s_k(T)$ are solutions of the equations

$$d(s_k+\varphi S_k)/dT=-\pi^2k^2s_k,\qquad \varphi dS_k/dT=ns_k-mS_k \tag{30}$$

for initial conditions

$$s_k(0)=s_k^0=2\int_0^1K(z)\cos(\pi kz)\,dz,\quad S_k(0)=S_k^0=2\int_0^1L(z)\cos(\pi kz)\,dz, \tag{31}$$

which can easily be proved by using the orthogonality of the functions 1, $\cos(\pi z)$, $\cos(2\pi z)$... on the interval [0, 1]. The problem of Eqs. (30), (31) has a solution

$$\begin{aligned}s_k(T)&=\exp(p_1^kT)[m(s_k^0+\varphi S_k^0)+\varphi s_k^0p_1^k]/\Delta(p_1^k)+\\&+\exp(p_2^kT)[m(s_k^0+\varphi S_k^0)+\varphi s_k^0p_2^k]/\Delta(p_2^k),\\S_k(T)&=\exp(p_1^kT)[ns_k^0+\varphi S_k^0(p_1^k+\pi^2k^2+n)]/\Delta(p_1^k)+\\&+\exp(p_2^kT)[ns_k^0+\varphi S_k^0(p_2^k+\pi^2k^2+n)]/\Delta(p_2^k),\end{aligned} \tag{32}$$

where

$$\begin{aligned}p_{1,2}^k&=\{-(m+\varphi\pi^2k^2+n\varphi)\pm[(m+\varphi\pi^2k^2+n\varphi)^2-4m\varphi\pi^2k^2]^{1/2}\}/2\varphi,\\\Delta(p_{1,2}^k)&=2\varphi p_{1,2}^k+m+\varphi n+\varphi\pi^2k^2.\end{aligned} \tag{33}$$

It can easily be proved that for $\varphi>0$, $m>0$, $n>0$ the numbers $p_{1,2}^k$ will always be real and negative, i.e., the terms of the sum in Eq. (28) decrease with time. Thus, in this special case one can calculate B(0), and hence the problem of the second approximation with respect to Peclet number Pe can be considered formulated. The function w(a, c) can be quite complex in form, and calculation of the integral in Eq. (27) will be accompanied by definite difficulties. The situation simplifies significantly when the material is uniformly distributed within the porous body at the initial moment, i.e., $K(z)=K_0=\text{const}$, $L(z)=L_0=\text{const}$, which is often the case in practical problems. In this case all s_k^0, , S_k^0 vanish, the series in the expressions of Eq. (28) vanish, the functions g_0, G_0 are simple exponentials,

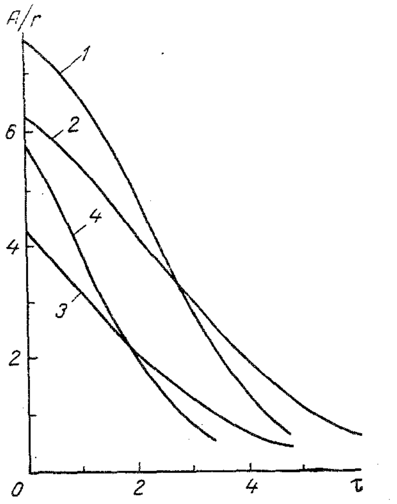

Fig. 1. Kinetics of impurity extraction from the flow region of a porous precipitate for $\varepsilon = 0.5$: 1) $r = 0.5$ and $b = 1$; 2) 0.5 and 2; 3) 1 and 2; 4) 1 and 1.

Eq. (29), and in particular, at $nK_0 = mL_0$ they are constants, in which case (which is also quite widespread) the integral with w in Eq. (27) becomes equal to zero. We note that computations are also significantly simplified for a nonlinear isotherm, if $K(z) = K_0 = \text{const}$, $L(z) = L_0 = \text{const}$, since in this case Eqs. (17), (18) and conditions (19), (4) are satisfied by functions g_0 and G_0 which depend solely on T. We have

$$T=\frac{\varphi}{n}\int\limits_{L_0}^{G_0} dx/[M-\varphi x-f(x)],\quad g_0=M-\varphi G_0 \tag{34}$$

and, just as before, if K_0 and L_0 are related by the sorption isotherm $K_0 = f(L_0)$, the integral in Eq. (27) vanishes, since the solution of the problem of Eqs. (17)-(19), (4) will be the functions $g_0 = K_0$ and $G_0 = L_0$.

When the initial functions depend on z it is convenient to seek the solution of the problem of Eqs. (17)-(19), (4) by the Galerkin method in the form of Eq. (28). The first equation of Eq. (30) retains its form, while the second is replaced by:

$$\begin{gathered}\varphi dS_k/dT=n\,[s_k-F_k(S_0,\ S_1,\ \ldots\ S_N)],\\ F_k=\beta_k\int\limits_0^1 f\left[S_0+\sum_{i=1}^{N}S_i(T)\cos(\pi zi)\right]\cos(\pi zk)\,dz,\\ k=0,1,\ 2,\ \ldots,\ N;\quad \beta_0=1,\quad \beta_k=2,\quad k=1,\ 2,\ \ldots,\ N,\end{gathered} \tag{35}$$

where N is the number of functions $\cos(\pi z)$, $\cos(2\pi z)$, used in the Galerkin method. The initial conditions for the nonlinear system (35) are defined by Eq. (31) with corresponding change in normalization at $k = 0$. The system of equations obtained has an integral $s_0 = M - \varphi S_0$, so that for a given N there are in fact $2N + 1$ independent equations.

Thus, basic equations and initial conditions have been obtained for the first two functions of the external expansion in the small parameter Pe for cases of practical interest. The problem for the zeroeth approximation $f_0(z, \tau) = A(\tau)$ reduces to solution of Eq. (12) with an additional condition defined by Eq. (24):

$$A'(\tau)+A(\tau)+\varphi\{f^{-1}[A(\tau)]\}'_\tau=w\{f^{-1}[A(\tau)],\ A(\tau)\}+R(\tau), \tag{36}$$

$$\varphi f^{-1}[\mathrm{A}(0)]=\int\limits_0^1[K(z)+\varphi L(z)]\,dz-A(0). \tag{37}$$

The concentrations of material in the stagnant zone and flow region are related by the sorption isotherm: $F_0(\tau) = f^{-1}[A(\tau)]$. To find the first approximation functions f_1 and F_1, defined by Eq. (11), the main unknown function is $B(\tau)$ is considered known. The initial condition can be found with Eq. (27). Only at w = c does this expression permit direct determination of B(0), while in other cases it is necessary to calculate an integral containing the zeroeth approximation function for the "internal" solution G_0, g_0. The functions G_0 and g_0 are calculable in two cases: 1) for a linear isotherm f = ma/n, where the solution is defined by Eqs. (28)-(33); 2) for the special (but quite widespread) initial conditions $K(z) = K_0 = \text{const}$, $L(z) = L_0 = \text{const}$. In these cases the functions g_0 and G_0 are defined by Eq. (34), the integral in Eq. (27) becomes single, and when the initial concentrations are related by the equilibrium sorption isotherm $K_0 = f(L_0)$ in general vanishes. In the general case g_0 and G_0 can be found by the Galerkin method in the form of Eq. (28) using Eq. (35) and the first expressions of Eq. (30). It should be noted that the proposed method can be applied to more complex cases with no complications in principle, for example, by taking the sorption isotherm in a more general form, considering the possibility of dependence of w on z in explicit form, in addition to a and c. In the latter case Eq. (12) is replaced by:

$$A' + A + \varphi\{f^{-1}[A]\}'_\tau = R(\tau) + \int_0^1 w\{f^{-1}[A(\tau)],\ A(\tau),\ z\}\,dz. \qquad (38)$$

As an example of application of the proposed method, we will consider the first approximation of washing of a precipitate with f(a) being a Langmuir-type dependence [3]:

$$f(a) = ra/(a + b). \qquad (39)$$

We take the functions K and L equal to unity, and will not consider chemical reaction in the precipitate, w = 0, and take R = 0, i.e., at the input to the precipitate a pure washing liquid is supplied. In this case the quantity A(0) is given by

$$A(0) = 0{,}5\,\{(1/\varepsilon + \varphi b + r) - [(1/\varepsilon + \varphi b + r)^2 - 4r/\varepsilon]^{1/2}\}, \qquad (40)$$

easily derivable from Eq. (24), and the solution of Eq. (12) is

$$\tau = \ln\left[\frac{A(0)}{A}\right] + \frac{\varphi b\,[A(0) - A]}{(r - A)\,[r - A(0)]} + \frac{\varphi b}{r}\ln\left\{\frac{A(0)\,(r - A)}{A\,[r - A(0)]}\right\}. \qquad (41)$$

Some curves describing Eq. (41) for various parameter values are shown in Fig. 1. The function B(τ) is found in a similar manner, but will not be presented because of its cumbersomeness. The internal solution of the zeroth approximation in Pe is found with quadrature (34), which with our isotherm (39) can be expressed in terms of elementary functions.

NOTATION

a, c, dimensionless material concentrations in stagnant zones and flow region of porous body; D, dispersion coefficient; f_i, F_i, external variables; g_i, G_i, external variables; k, mass exchange coefficient between stagnant zones and flow region; K(z), L(z), dimensionless initial material concentrations in flow and stagnant zones; l, precipitate thickness; Pe = u/D, Peclet number; T = τ/Pe, internal time; t, x, dimensionless time and coordinate along layer; u, filtration rate; $z = x/l$, dimensionless coordinate; ε, fraction of flow region in total void volume of porous body; τ, external time.

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METHOD FOR CALCULATING CONVECTIVE DRYING OF MOIST MATERIALS

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The dependence of output of a drying apparatus in terms of evaporated moisture is obtained as a function of time and parameters and properties of the material being dried. Calculation results are compared with experiment.

The basic quantity which characterizes operation of drying apparatus is the quantity of water evaporated, which is usually determined from material balance equations. A method which would permit determination of this quantity from known or easily measured characteristics of the process and relate the engineering calculation of drying equipment output to the kinetics of drying is of practical interest.

We will write the combined equation of thermal balance and heat exchange for the operating conditions of a convective drying apparatus

$$[\alpha_{\mathrm{cr}}(T_{\mathrm{h}}-T_{\mathrm{w}})+\bar{\alpha}(T_{\mathrm{h}}-T_{\mathrm{s}})]F\tau=rm_0N\tau_{\mathrm{I}}+rm_0\frac{d\bar{u}}{d\tau}\tau_{\mathrm{II}}+(c_0m_0+c_{\mathrm{m}}m_{\mathrm{m}}^{\mathrm{II}})\frac{dT_{\mathrm{mt}}}{d\tau}\tau_{\mathrm{II}}. \tag{1}$$

In accordance with the definition of the Stenton number, this parameter in the first and second drying periods can be written in the following manner:

$$\mathrm{St}_{\mathrm{cr}}=\frac{\alpha_{\mathrm{cr}}}{c_pv\rho}=\frac{T_1-T_2}{T_{\mathrm{c}}-T_{\mathrm{w}}}\frac{f}{F};\quad \overline{\mathrm{St}}=\frac{\bar{\alpha}}{c_pv\rho}=\frac{T_1-T_2}{T_{\mathrm{h}}-T_{\mathrm{s}}}\frac{f}{F}. \tag{2}$$

With consideration of Eq. (2), we write Eq. (1) in the form

$$\left(1+\frac{\overline{\mathrm{St}}}{\mathrm{St}_{\mathrm{cr}}}\frac{T_{\mathrm{h}}-T_{\mathrm{s}}}{T_{\mathrm{h}}-T_{\mathrm{w}}}\right)\tau=\frac{1}{c_pv\rho f(T_1-T_2)}\left[rm_0N\tau_{\mathrm{I}}+rm_0\frac{d\bar{u}}{d\tau}\tau_{\mathrm{II}}+(c_0m_0+c_{\mathrm{m}}m_{\mathrm{m}}^{\mathrm{II}})\frac{dT_{\mathrm{mt}}}{d\tau}\tau_{\mathrm{II}}\right]. \tag{3}$$

According to [1], for convective drying

$$\frac{\overline{\mathrm{Nu}}}{\mathrm{Nu}_{\mathrm{cr}}}\equiv\frac{\overline{\mathrm{St}}}{\mathrm{St}_{\mathrm{cr}}}=(1+\mathrm{Rb})N^{*0,57};\quad \frac{T_{\mathrm{h}}-T_{\mathrm{s}}}{T_{\mathrm{h}}-T_{\mathrm{w}}}=N^{*0,43}. \tag{4}$$

The drying rate in the first period can be expressed as

$$N=\frac{m_{\mathrm{m}}^{\mathrm{I}}}{m_0\tau_{\mathrm{I}}}. \tag{5}$$

From the definition of the Rebinder number Rb, it follows that

$$\frac{dT_{\mathrm{mt}}}{d\tau}=\frac{d\bar{u}}{d\tau}\frac{r}{c}\mathrm{Rb}. \tag{6}$$

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